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A new power series solution on the electrostatic pull-in instability of nano cantilever actuators

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Abstract

In this paper, a power series solution is used to study the deflection and pull-in instability of nano cantilever electromechanical switches using a distributed parameter model. In the modeling of intermolecular force the Casimir force and in the modeling of electrostatic force, the fringing field effect is taken into account. A closed form power series solution basis on symbolic computation is introduced to obtain a semi analytical solution for the distributed parameter model. The minimum initial gap and the detachment length of the actuator that does not stick to the substrate due to the Casimir attractions, as an important parameter in pull-in instability of a nano cantilever actuator, is calculated by obtained power series. Furthermore, the present method is capable of determining stress distribution of the nano beam actuator at the onset of instability. It is seen that the Casimir effect significantly reduce the maximum value of stress resultants at the onset of instability. The obtained results are compared with numerical results and the Adomian decomposition method. The obtained solution in compare with the Adomian decomposition method represents a remarkable accuracy.

Keywords: Nano; Pull-in; Cantilever; Electromechanical Switches; Power Series.

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1. Introduction

Conductive cantilever nano-actuators are one of the common components in developing nano-electromechanical system (NEMS) switches of nano technology [1, 2]. A typical form of NEMS actuator is a nano-beam which is suspended above a conductive flat ground (substrate). Applying voltage difference between the nano-beam and the ground plane causes the nano-beam to deflect downward and be attracted toward to the substrate. At a critical deflection/voltage, which is known as pull-in deflection/voltage, the nano-beam pulls-in onto the substrate and the instability occurs [3]. The inter-molecular forces significantly influence the instability of nano-beam, at nano-scale separations [2]. When the separation is large enough (typically above 20 nm), the retardation is appeared. In this case, the intermolecular interaction between nano beam surface and substrate surface can be described by the Casimir force [4-6]. Considering the ideal case, the Casimir interaction is proportional to the inverse fourth power of the separation [9]. Some researchers [10-14] studied the effect of Casimir force on the instability of NEMS. Design of reliable NEMS requires crucial knowledge about the mechanical stress field in the structure [15, 16]. If a nano-beam is not strong enough to bear internal stresses, it might deform or break before instability occurs. Unfortunately, all mentioned investigators assumed the electrostatic and intermolecular forces uniform along the beam and therefore could not evaluate the internal stress resultants i.e. internal shear force and bending moment along the beam [7-14]. Ramezani et al. [17, 18] applied distributed parameter model to study the instability of nano-cantilevers. However, their solution did not satisfy all boundary conditions and consequently could not determine the stress resultants. These shortcomings have been overcome in our previous work [2], which considered the effect of Casimir attractions on the electrostatic pull-in instability of nano-actuators using Adomian decomposition method. Although, the results of Adomian decomposition method in compare with numerical results are acceptable, but their accuracy is not perfect.

This shortcoming on the accuracy of Adomian decomposition method has been overcome in present study using power series method. In this study, power series method is introduced as a new approach to study the pull-in behavior and internal stress resultants of nano-actuator using a distributed parameter model. A fair comparison is made between the presented method and Adomian decomposition method and numerical results.

2. Mathematical model

Figure 1 shows a nano-cantilever beam, of length L with uniform rectangular cross section of thickness h and width w. the initial gap between the movable beam and the ground plane is g. The constitutive material of the nano-cantilever is assumed linear elastic and only the static deflection of the nano-beam is considered. The effect of finite kinematics is negligible when $L > 10g$ [2], and hence, finite kinematics is not considered. This simplification is acceptable for most cases [17]. Considering the first order fringing field correction, the electrostatic force per unit length of nano-cantilever, (electrical force) can be defined as [19]

$$f_{elec} = \frac{\varepsilon_0 w V^2}{2(g-y)^2}\left(1 + 0.65\frac{(g-y)}{w}\right) \tag{1}$$

where $\varepsilon_0 = 8.854\times10^{-12}\ c^2/Nm^2$ is the permittivity of vacuum, V is the applied external voltage and y is the deflection of the beam. The intermolecular forces per unit length of the beam (f_{mole}) including the Casimir force is defined as [2-4]

$$f_{mole} = \frac{\pi^2 \hbar c w}{240(g-y)^4} \tag{2}$$

where $\hbar = 1.055 \times 10^{-34}\ Js$ is Planck's constant divided by 2π, and $c = 2.998 \times 10^8\ m/s$ is the light speed. When cantilevers are sufficiently wider than the separation space, (2) provides acceptable results [2,3].

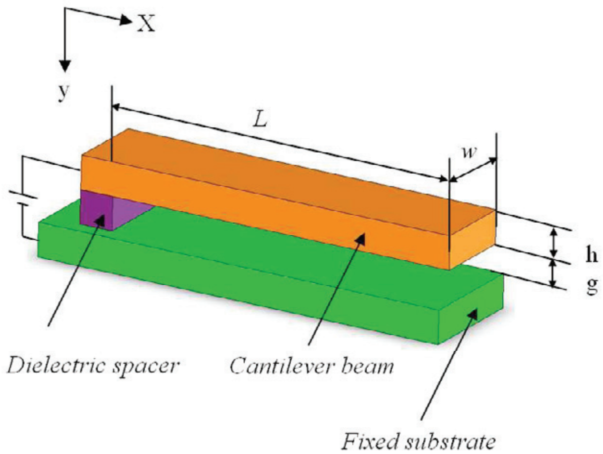


Fig.1. Schematic representation of a cantilever nano-beam

Therefore only the cantilevers that are wider than the separation $g/w \le 1$ are considered in this study. The appropriate approximation of the beam deflection can be found by applying the virtual work principle. In the absence of non-conservative forces and by considering only the static elastic small deflection of the nano cantilever beam, we can write

$$\delta W = \delta E_{elast} - \delta W_{elec} - \delta W_{mole} = \int_0^L \left(E_{eff} I \frac{d^2 y}{dX^2} \delta \frac{d^2 y}{dX^2} - f_{elec} \delta y - f_{mole} \delta y \right) dX \tag{3}$$

After integrating (3) we have

$$\delta W = E_{eff} I \frac{d^2 y}{dX^2} \delta \frac{dy}{dX} \Big|_0^L - E_{eff} I \frac{d^3 y}{dX^3} \delta y \Big|_0^L + \int_0^L \left(E_{eff} I \frac{d^4 y}{dX^4} - f_{elec} - f_{mole} \right) \delta y \, dX \tag{4}$$

As there are no deflection and rotation at the fixed end and also due to the absence of the bending moment and shear force at the free end of the beam, the boundary value problem for a cantilever nano-beam can be defined as

$$E_{eff} I \frac{d^4 y}{dX^4} = f_{elec} + f_{mole} \tag{5-a)}$$

where the geometrical boundary conditions at fixed end are

$$y(0) = y'(0) = 0 \tag{5-b)}$$

And natural boundary conditions at free end are

$$y''(L) = y'''(L) = 0 \tag{5-c)}$$

where y is the deflection of the beam, X is the position along the beam measured from the clamped end, prime denotes differentiation with respect to X. E_{eff} is the effective Young's modulus which is equal to $wh^3/12$, and I is the moment of inertia of the beam cross section [20] . For convenience, the model is parameterized in the nondimensional form. Substituting (1) and (2) into (5) and introducing the nondimensional variables

$$\alpha = \frac{\pi^2 h c w L^4}{240 g^5 E I} \quad \beta = \frac{\varepsilon_0 w V^2 L^4}{2g^3 E I} \quad \gamma = 0.65 \frac{g}{w} \quad x = \frac{X}{L} \quad u = \frac{y}{g} \tag{6)}$$

leads to the following nondimensional form

$$\frac{d^4 u}{dx^4} = \frac{\alpha}{(1-u(x))^4} + \frac{\beta}{(1-u(x))^2} + \frac{\gamma\beta}{(1-u(x))} \tag{7}$$

subject to the following conditions

$$u(0) = u'(0) = 0, \quad at \quad x = 0 \quad and \quad u''(1) = u'''(1) = 0, \quad at \quad x = 1 \tag{8}$$

where α, β and γ correspond to the values of intermolecular force, applied voltage and fringing field respectively.

3. Power Series

A differential–algebraic equation (DAE) can be considered as the equations are an exception to the prescribed

$$F(y, y', x) = 0, \tag{9}$$

with initial condition

$$y(a) = y_0 \tag{10}$$

where F and y are vector functions. The solutions of (9) can be assumed such that

$$y = y_0 + ex \tag{11}$$

where e is a vector function. By substituting (11) into (10) and neglecting the higher order term, we have the linear equation of e in the form

$$M\,e = N \tag{12}$$

where M and N are constant matrixes. By solving this Equation (i.e. 12), the coefficients of e in (11) can be determined. Repeating the above procedure for higher terms, we can get the arbitrary order power series of the solutions for (9). For more details the readers are refereed to [21 - 24].

4. Analytical Solution

Equation (7) can be written as the following set of first order nonlinear initial value differential equations

$$u_1'(x) = u_2(x),\ u_2'(x) = u_3(x),\ u_3'(x) = u_4(x),\ u_4'(x) = \frac{\alpha}{(1-u_1(x))^4} + \frac{\beta}{(1-u_1(x))^2} + \frac{\gamma\beta}{(1-u_1(x))} \tag{13-a}$$

$$u_1(1) = A,\ u_2(1) = B,\ u_3(1) = 0,\ u_4(1) = 0 \tag{13-b}$$

subject to the following constraints

$$u_1(0) = 0, \qquad u_2(0) = 0 \tag{14}$$

where undetermined coefficients A and B correspond to the deflection and first derivative of beam deflection with respect to x at x=0, respectively. These new coefficients will be determined later by using the boundary conditions at x=0 i.e. (14). Therefore, by using power series technique, the polynomial solution of (13) is obtained which can be summarized to (15). In order to verify the convergence of obtained series, the deflection of a typical nano-actuator is computed analytically and the solutions are compared with the numerical data. Numerical results are obtained using a combination of trapezoid as base scheme and Richardson extrapolation as enhancement scheme. The step size of the parameter variation is selected based on the sensitivity of the parameter to the tip deflection. Table 1 presents the variation of the cantilever tip deflection (u_{tip}), with respect to dimensionless length of the beam (x) using different selected terms in the series. This table ensures the convergence of the results. As it is seen, higher accuracy can be obtained by evaluating more terms of the solution *u(x)*.

$$\begin{aligned} u_1(x) = A + B(x-1) &- \frac{\left((A-1)^2(A\gamma-1-\gamma)\beta-\alpha\right)(x-1)^4}{24(A-1)^4} + \frac{\left((A-1)^2(A\gamma-2-\gamma)\beta-4\alpha\right)B(x-1)^5}{120(A-1)^5} \\ &- \frac{\left((A-1)^2(A\gamma-3-\gamma)\beta-10\alpha\right)B^2(x-1)^6}{360(A-1)^6} + \frac{\left((A-1)^2(A\gamma-4-\gamma)\beta-20\alpha\right)B^3(x-1)^7}{840(A-1)^7} + P(x-1)^8 + \ldots \end{aligned} \tag{15-a}$$

where P is

$$P = \frac{\beta^2\gamma^2}{40320(A-1)^3} - \frac{\beta^2\gamma}{13440(A-1)^4} + \frac{\beta\left(12\gamma B^4+\beta\right)}{20160(A-1)^5} - \frac{\beta\left(24B^4+\alpha\gamma\right)}{8064(A-1)^6} + \frac{\alpha\beta}{6720(A-1)^7} - \frac{B^4\alpha}{48(A-1)^8} + \frac{\alpha^2}{10080(A-1)^9} \tag{15-b}$$

Table 1.The variation of the tip deflection of a typical beam with respect to *x* obtained using different selected terms of power series for α=0.3, β=0.3 , and *g*/*w* = 1: The analytical solution converges to the numerical solution as the number of the selected terms increase.

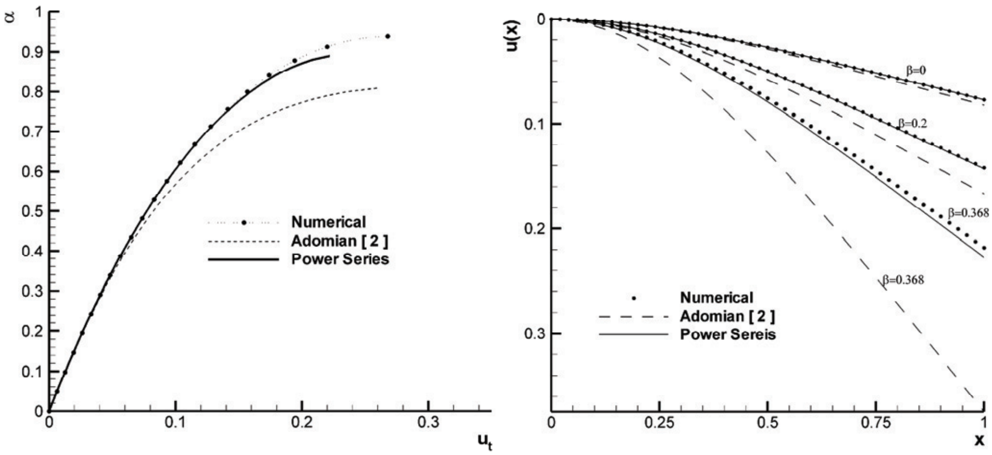
Terms	*Tip Deflection Power Series*	*Tip Deflection Adomian [2]*	*Power Series Errors*	*Adomian Errors*
4 Terms	0.0111	0.0788	0.0163	0.0157
6 Terms	0.0956	0.1039	0.0011	0.0094
7 Terms	0.0954	0.0890	0.0009	0.0103
8 Terms	0.0959	0.099	0.0014	0.0054
9 Terms	0.0957	0.0909	0.0012	0.0036
10 Terms	0.0957	0.0972	0.0012	0.0027
Numerical	0.0945			

By using eight terms of power series, the global error between analytical and numerical results is less than 1.5%. Comparing this error with (the same series size of) Adomian method (5.7%) shows the power series method could compute deflection of nano-beam with more accuracy than Adomian method. The result of eight terms of power series with 1.5% error is within the acceptable range for most engineering applications. Therefore, eight terms are selected in the following section for the convenience.

5. Instability study

In order to study the instability of the nano-actuator, equation (7) is solved numerically simulated and the results are compared with that of equation (15). For any given α, β and *g/w*, the cantilever tip pull-in deflection can be obtained from equation (15) by setting du(1)/dβ → ∞. No physical solution exists for u

by increasing β beyond β_{PI}. For freestanding cantilevers, pull-in deflection can be obtained by setting $du(1)/d\alpha \rightarrow \infty$ in equation (15). In addition, the obtained results are compared with those of Adomian decomposition method proposed in [2].



(a) (b)

Fig. 2.(a): Relationship between α and the cantilever tip deflection when no voltage is applied. Collapse occurs for α values above the critical value of α=0.894. (b): Deflections of the cantilever for different values of β when α = 0.5 and g/w = 1. Collapse occurs when β reaches values greater than its critical.

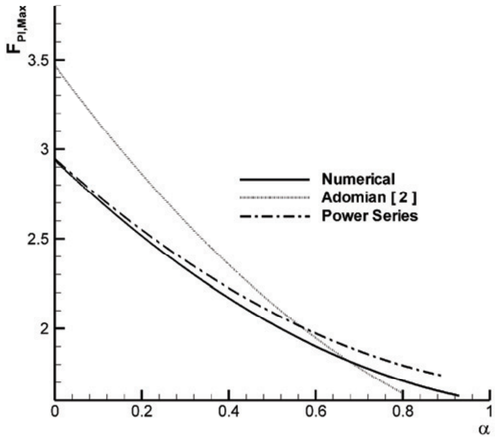
5.1. Intermolecular force at nanoscale separations

When the gap between the fixed and movable beams is small enough, the movable beam might collapse onto the substrate without applying voltage due to the intermolecular force. The relations between α and u_{tip} in the absence of voltage difference (freestanding) are presented in Fig. (2-a) When α exceeds the critical value α_{CP}, no solution exists for u_{tip} and the instability occurs even without any applied voltage. The maximum length of the actuator that does not stick to the fixed ground plane without applying voltage on the switch (L_{max}), is called the detachment length [17]. Alternatively, for a known switch length, there is a minimum gap, (g_{min}), between the switch and the substrate to ensure that the switch does not adhere to the substrate as a result of intermolecular force [17]. The detachment length and minimum gap of the actuator are basic design parameters for MEMS/NEMS and can be obtained by the critical value of α, i.e. α_{CP}. Substituting the value of α_{CP} into the definition of α (i.e. 6); the detachment length and minimum gap are obtained as

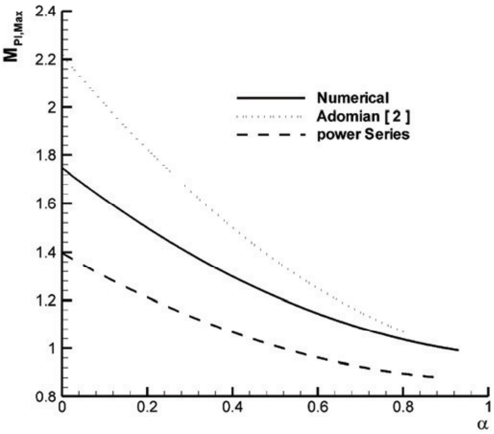
$$L_{\max} = \sqrt[4]{\frac{17.88 g^5 E_{eff} h^3}{\pi^2 \hbar c}}, \qquad g_{\min} = \sqrt[5]{\frac{L^4 \pi^2 \hbar c}{17.88 E_{eff} h^3}} \tag{16}$$

5.2. Electrostatic and intermolecular force at nanoscale separations

Figure (2-b) shows the centerline deflection of a typical nano-beam under both electrostatic and intermolecular loading when β increases from zero to instability point. This figure reveal that the beam have initial deflection due to the presence of intermolecular force even when no voltage is applied (β=0). As seen, the power series solution in compare with the same size of Adomian series solution (i.e. that on [2]) is more powerful to predict the deflection and instability of nano-cantilever beams.



(a)



(b)

Fig. 3. Effects of the intermolecular force on the (a): $F_{PI,max}$ of cantilevers (b) $M_{PI,}$max of cantilevers

6. Stress resultants

Design of reliable NEMS requires crucial knowledge about the distribution of internal stress over the length of the beam. The maximum value of shear stress and bending stress at the onset of instability are very important as the most critical state of stress in the engineering applications. Based on Euler beam theory, these parameters can be directly computed from stress resultants [20]. In order to determine critical values of stress resultants, we define $F_{PI,max}$ and $M_{PI,max}$ as the dimensionless maximum value of the shear force and bending moment at the onset of instability, respectively as follows

$$F_{PI,\max} = \frac{F_0 L^3}{E_{eff} Ig}, \qquad M_{PI,\max} = \frac{M_0 L^2}{E_{eff} Ig} \tag{17}$$

where F_0 and M_0 are shear force and bending moment at the cross-section of the beam fixed end ($x=0$). By these definitions, $M_{PI,max}$ and $F_{PI,max}$ equal to u'' ($x=0$) and $-u'''$ ($x=0$), respectively [20]. Fig. 3 show the effects of fringing field for $g/w=1$ and neglecting the effect of intermolecular forces on $F_{PI,max}$ and $M_{PI,max}$ of the cantilever respectively. As seen, intermolecular force decreases the values of $F_{PI,max}$ and $M_{PI,max}$ of the beam. Since intermolecular force reduces the pull-in deflection of the beam, it decreases the maximum value of stress resultants at the onset of instability. On the other hand, fringing field increases $F_{PI,max}$ and $M_{PI,max}$ of the cantilever. It can be observed that the proposed power series solution is more accurate than the Adomian one in compare with numerical results. In addition, power series solution produces better results for u_{PI} compared to the proposed Adomian results ([2]).

7. Conclusions

Pull-in parameters and internal stress field of nano-cantilevers were computed using power series method. The power series closed-form solution satisfies all boundary conditions and no initial guess is required for solving the problem. It is found that intermolecular forces decrease the pull-in deflection and voltage of nano-beams. The minimum initial gap and detachment length of freestanding nano-actuator were determined which are the useful design parameters for nano-electromechanical switches. We also compared the presented analytical solution with the Adomian solution and the numerical results. The power series solution overcame the shortcomings of the lumped parameter model in underestimating the pull-in voltage of nano-beams, and also has more accuracy in compare with the Adomian method.

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